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ABSTRACT

Literature at national and international levels argues the importance of including mental computation in a mathematics curriculum that promotes number sense. However, mental computation does not feature in importance in the current Queensland mathematics syllabus documents. Hopefully, with the writing of a new mathematics syllabus, mental computation will feature with more prominence. It has been posited that when children are encouraged to formulate their own mental computation strategies, they learn how numbers work, gain a richer experience in dealing with numbers, and develop number sense. In the literature, a wide variety of addition and subtraction mental strategies has been identified and characteristics of good mental computers have been documented. These findings are useful to inform teachers of children's thinking, and help them better understand children's explanations. However, little research has attempted to explain why or how children develop these strategies and why some children are proficient. Thus, the intention of present study was to go beyond reporting the existing situation in schools to investigating, in depth, associated factors, and to develop a comprehensive model for mental computation. This paper reports a study of Year 3 children's addition and subtraction mental computation abilities, and the complexity of interaction of cognitive, metacognitive, and affective factors that supported and diminished their ability to compute efficiently. As well, the part memory plays in mental computation was investigated. Finally, some implications for teaching are discussed. (Contains 46 references.) (Author/ASK)

# Mental computation: Is it more than mental architecture?

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Mental computation: Is it more than mental architecture?

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*Literature at national and international levels argues the importance of including mental computation in a mathematics curriculum that promotes number sense. However, mental computation does not feature in importance in the current Queensland mathematics syllabus documents. Hopefully, with the writing of a new mathematics syllabus, mental computation will feature with more prominence. It has been posited that when children are encouraged to formulate their own mental computation strategies, they learn how numbers work, gain a richer experience in dealing with numbers, and develop number sense. In the literature, a wide variety of addition and subtraction mental strategies has been identified and characteristics of good mental computers have been documented. These findings are useful to inform teachers of children's thinking, and help them better understand children's explanations. However, little research has attempted to explain why or how children develop these strategies and why some children are proficient. Thus, the intention of present study was to go beyond reporting the existing situation in schools to investigating, in depth, associated factors, and to develop a comprehensive model for mental computation. This paper reports a study of Year 3 children's addition and subtraction mental computation abilities, and the complexity of interaction of cognitive, metacognitive, and affective factors that supported and diminished their ability to compute efficiently. As well, the part memory plays in mental computation was investigated. Finally, some implications for teaching are discussed.*

The architecture of mental addition and subtraction was initially described at the Australian Association for Research in Education Conference in 1997 (Heirdsfield & Cooper, 1997). Three studies, each building on the previous one, were reported, and as a result, the mental architecture of a proficient mental computer was described. In particular, the third study found a complex interaction among factors that appeared to be connected with proficient mental computation. The third study (reported in Heirdsfield & Cooper, 1997) constituted a small part of a study, whose main purpose was to develop an explanation of why some children are better at addition and subtraction mental computation than others. Mental computation is defined as "the process of carrying out arithmetic calculations without the aid of external devices" (Sowder, 1988, p. 182). The research was undertaken in response to claims that addition and subtraction mental computation should be included in number strands of mathematics curricula. This paper reports the findings of this larger study.

Addition and subtraction mental computation, as defined in this study, is not mentioned in the existing Queensland curriculum document, *Years 1 to 10 mathematics teaching, curriculum and assessment guidelines* (Department of Education, Queensland, 1987a). Nevertheless, in some of the support documents, *Years 1 to 10 mathematics sourcebooks* (e.g., Department of Education, Queensland, 1987b, 1988, 1990), specific mental computation strategies are mentioned. It is of interest to note that the mental strategies for two-digit addition that are mentioned in the Year 7 sourcebook are taught to

Dutch children in second grade (Beishuizen, 1993). Further, these same strategies were reported as being employed spontaneously by children as young as 8 and 9 years old in Brisbane schools, where addition and subtraction mental strategies are not taught at this age (Cooper, Heirdsfield, & Irons, 1996).

Literature at national and international levels argues the importance of including mental computation in a mathematics curriculum that promotes number sense (e.g., Klein & Beishuizen, 1994; McIntosh, 1998; Reys, Reys, Nohda, & Emori, 1995; Sowder, 1992; Verschaffel & De Corte, 1996; Willis, 1992). It has been posited that when children are encouraged to formulate their own mental computation strategies, they learn how numbers work, gain a richer experience in dealing with numbers, develop number sense, and develop confidence in their ability to make sense of number operations (Kamii & Dominick, 1998; Reys & Barger, 1994; Sowder, 1990).

Carroll (1997) and Kamii, Lewis, and Livingston (1993) documented the mental and written computational procedures invented by children who are active in their learning. They showed that children could produce a wide variety of efficient strategies that exhibited sound number understanding even though there was little direct teaching of algorithms. They also found that the active development of knowledge encouraged children to participate in the construction of problems and the explanation of solution strategies.

In Queensland, there are few reform classrooms and the traditional pen and paper algorithms are still taught out of context and in situations where children have little or no input into constructing problems and explaining solutions. As Cooper, Heirdsfield and Irons (1995 & 1996) reported, this has resulted in a tendency for Queensland children to use strategies for mental computation that reflect the procedures underlying the pen and paper algorithms regardless of their knowledge and ability to use more efficient strategies. However, there is evidence that even children in these situations can and do employ a variety of computational methods, particularly before instruction in the traditional pen and paper algorithms (Heirdsfield, 1999).

The purpose of the research was to develop an explanation why some children are better at addition and subtraction mental computation than others. In particular, a fundamental aim was to identify factors and the relationship between factors which influence children's proficiency in addition and subtraction mental computation. In order to commence this study, the literature was consulted to identify some possible factors on which to base the initial investigation.

The literature has shown that mental computation may be viewed as a subset of number sense, as students who exhibit proficiency in mental computation also display number sense (e.g., McIntosh, 1996; McIntosh, Reys, & Reys, 1992; Sowder, 1990, 1992). Research on mental computation has proposed specific connections among mental computation and aspects of number sense, in particular, number facts knowledge and estimation (e.g., Heirdsfield, 1996; Sowder, 1992). Other research relating to computation (in particular, children's natural strategies) has reported connections with number and operation and numeration, for example, place value, (e.g., Kamii, Lewis, & Jones, 1991).

Further, relationships have been posited between mental computation and affective factors, for example, beliefs (about mathematics or a particular domain of mathematics) and beliefs about oneself (self-efficacy) (DeBellis & Goldin, 1997). Such beliefs as mathematics should make sense, there are often different ways of solving problems, and there may be more than one answer would result in a quite different performance from that where mathematics is viewed as set of rules to be learnt and it need not make sense. Beliefs about the nature of mathematics could be manifested in a student's disposition - mastery orientation or performance orientation (Prawat, 1989). In relation to computation, mastery oriented students would aim for understanding and flexibility, and where monitoring, checking, and planning may be evident; whereas, performance oriented students would tend to aim to complete a task as quickly as possible, and not attend to understanding and reflection.

Hope (1987), Hope and Sherrill (1987), Reys (1985), and Sowder (1994) identified characteristics of proficient mental computers. Skilled mental computers used a variety of strategies in different situations (depending on numbers and context), because they were disposed to making sense of mathematics (Sowder, 1994). Therefore, they were aware of a variety of strategies and had the confidence to use them. There was also evidence of reflection and regulation. Further, Hope (1987) and Dowker (1990) reported children and adults choosing strategies based on their knowledge of number and operations, and choosing appropriate strategies to deal with the problems.

In summary, research on mental computation and number has proposed connections among mental computation and number sense, particularly number facts, computational estimation, numeration, and properties of number and operation; social and affective issues including attributions, self efficacy, and social context (e.g., classroom and home); and metacognitive processes.

## The study

The research consisted of two studies, a pilot study and a main study. Both studies were based on interviews developed to investigate mental computation (strategies and accuracy) and other aspects that were identified from the literature. The findings of the pilot study informed the main study.

*Subjects*

The subjects were Year 3 students from two Brisbane Independent School that serve high and middle socio-economic areas. The students (thirteen in all) were selected (from a population of three Year 3 classes, 60 students in all) after participating in a structured mental computation selection interview. As proficiency in mental computation was defined in terms of both flexibility and accuracy, both these factors were considered when selecting the students. As a result of their performance on the selection items, students were identified as accurate and flexible, accurate and not flexible, inaccurate and flexible, and inaccurate and not flexible (see Table 1).

Table 1

*Students selected for Study*

| | Accurate | Inaccurate |
|---|---|---|
| Flexible | 4 | 3 |
| Inflexible | 2 | 4 |

*Instruments*

The students participated in indepth interviews, which addressed mental computation strategies, number facts, computational estimation, numeration, number and operations, and investigated metacognition, affect, beliefs and evidence of mental representations. These tasks have been described elsewhere (Heirdsfield & Cooper, 1997). As a result of analysing the pilot study, another factor, memory seemed to impact on mental computation. Therefore, memory tasks were also presented to the students. These addressed short term recall, short term retention, and executive planning.

*Interview procedures*

The students were withdrawn from class and participated in a series of videotaped semi-structured clinical interviews in a quiet room in the school. Although the same questions were to be presented to all subjects, questions were also contingent on responses (typical of a semi-structured interview approach). Reasons for deviating from the common tasks included: the student became agitated because of constant failure; the student's responses indicated that the tasks were too easy.

*Analysis*

For the purposes of identifying flexibility in mental computation, mental computation strategies were identified using the categorisation scheme presented in Table 2 (based on Beishuizen, 1993; Cooper, Heirdsfield, & Irons, 1996; Reys, Reys, Nohda, & Emori, 1995; Thompson & Smith, 1999).

Mental computation responses were analysed for strategy choice, flexibility, accuracy, and understanding of the effects of operation on number, numeration, computational estimation, and number facts. Analysis of the interviews investigating these individual factors was also undertaken, with the intention of exploring connections with mental computation. For the memory tasks, scores and strategies were recorded.

Table 2

*Mental Strategies for Addition and Subtraction*

| Strategy | | Example |
|---|---|---|
| *Counting* | | 28+35: 28, 29, 30, .. (count on by 1)<br>52-24: 52, 51, 50, .. (count back by 1) |
| *Separation* | *right to left (u-1010)*<br><br>*left to right (1010)*<br><br>*cumulative sum or difference* | 28+35: 8+5=13, 20+30=50, 63<br>52-24: 12-4=8, 40-20=20, 28 (subtractive)<br>:4+8=12, 20+20=40, 28 (additive)<br><br>28+35: 20+30=50, 8+5=13, 63<br>52-24: 40-20=20, 12-4=8, 28 (subtractive)<br>:20+20=40, 4+8=12, 28 (additive)<br><br>28+35: 20+30=50, 50+8=58, 58+5=63<br>52-24: 50-20=30, 30+2=32, 32-4=28 |
| *Aggregation* | *right to left (u-N10)*<br><br>*left to right (N10)* | 28+35: 28+5=33, 33+30=63<br>52-24: 52-4=48, 48-20=28 (subtractive)<br>: 24+8=32, 32+ 20=52, 28 (additive)<br><br>28+35: 28+30=58, 58+5=63<br>52-24: 52-20=32, 32-4=28 (subtractive)<br>: 24+20=44, 44+8=52, 28 (additive) |
| *Wholistic* | *compensation*<br><br>*levelling* | 28+35: 30+35=65, 65-2=63<br>52-24: 52-30=22, 22+6=28(subtractive)<br>24+26=50, 50+2=52, 26+2=28 (additive)<br><br>28+35: 30+33=63<br>52-24: 58-30=28 (subtractive)<br>22+28=50, 28 (additive) |
| *Mental image of pen and paper algorithm* | | Student reports using the method taught in class, placing numbers under each other, as on paper, and carrying out the operation, right to left. |

## Results

*Pilot study*

For the purposes of this paper, an overview of the findings of the pilot study will be reported. Little detail will be discussed. The four students in the pilot study were Clare (accurate and flexible), Mandy (accurate but inflexible), Emma (inaccurate and flexible), and Rosie (inaccurate and inflexible). Results for Clare and Mandy have been reported elsewhere (Heirdsfield, 1998; Heirdsfield & Cooper, 1997).

A picture of a proficient mental computer was starting to emerge. It appeared that a well-connected network of knowledge of the effects of operations on number, numeration, number facts, and computational estimation contributed towards flexibility and accuracy in mental computation. Other factors that appeared to contribute to mental computation were metacognitive strategies and beliefs, and beliefs about mathematics and self.

In the case of Mandy, who was accurate but not flexible, few links were made among the factors that were investigated; yet she was capable of holding many interim calculations in memory, resulting in overall accuracy. Mandy's number facts were fast and accurate (although it could be argued, not very efficient). Her number facts might have contributed to accuracy in mental computation. However, it is argued that her mental strategies would have taxed working memory. In contrast, Clare's mental strategies did not require such a load on working memory. Rather, memory was involved in making connections, for instance, remembering previously calculated number facts. On the other hand, many of Emma's errors were attributed to memory problems. Thus, memory seemed to impact on mental computation. To date, there is a paucity of studies investigating memory and mental computation, when mental strategies are not confined to mental images of pen and paper algorithms.

Two aspects of memory seemed to be significant: load on working memory while calculating, and retrieval from long-term memory of facts and strategies. Before commencing the main study, it was decided to review the literature concerning memory, and incorporate it in a model for investigation of mental computation.

Mental computation requires concurrent processing and temporary storage of information (holding interim calculations in memory), and retrieval of facts and strategies; that is, mental computation is cognitively demanding. Hunter (1978) suggested that the demand for retrieval of facts and strategies is met by long-term memory. In his study of expert mental calculators, Hunter posited that these experts not only build up vast resources of numerical equivalents (e.g., number facts and other more complicated numerical equivalents), but also a vast store of ingenious strategies. In this way, complex calculations can be handled more easily by accessing long-term memory for numerical equivalents and efficient calculative strategies, thus eliminating the need for massive calculations and demands on temporary storage. Further, Hunter suggested that the store of knowledge in long-term memory is acquired through an interest in numbers and strategies, not through "head-on memorisation" (p. 344).

However, his model did not account for concurrent processing of calculations, which occurs in working memory. A model for working memory consisting of several parts was proposed by Hitch (1978). Since then the model has been modified (eg, Baddeley, 1986, 1990, 1992; Baddeley & Logie, 1992; Logie, 1995) to include other components in working memory, In brief, the *central executive* component provided a processing function and a co-ordinating function, which included information organisation, reasoning, retrieval from long-term memory (access), and allocation of attention. The *phonological loop* (PL) was responsible for storage and manipulation of phonemic information, for instance, rehearsal of interim calculations. The *visuospatial scratchpad* (VSSP) dealt with holding and manipulating visuospatial information. This may involve representation of numbers in the head, or positional information of algorithms.

*Main study*

For ease of reporting, group numbers will be used to refer to specific categories of mental computers (see Table 3). Group 1 refers to those who were accurate and flexible, Group 2 to those who were accurate but not flexible, Group 3 to those who were inaccurate and flexible, and Group 4 to those who were neither accurate nor flexible.

Table 3

*Mental Computation Categories and Group Numbers*

| | Flexible | Not flexible |
|---|---|---|
| Accurate | 1 | 2 |
| Not accurate | 3 | 4 |

Fast and accurate number facts supported accuracy in mental computation. This would make sense, as fast and accurate recall of number facts from long term memory would result in less load on working memory, when more complex calculations are involved (as in mental computation of two- and three-digit addition and subtraction). Further, those students who scored poorly in the number facts test (slow and/or inaccurate) were inaccurate in mental computation. Thus, fast and accurate number facts are essential knowledge for accuracy in mental addition and subtraction.

In contrast, flexibility in mental computation was supported by number facts *strategies* . Students who were flexible in mental computation employed efficient number facts strategies (*derived facts strategies* ) in the number facts test. Some students (particularly those in Group 1) applied some number facts strategies to mental computation strategies. In the case of Group 3 students, using *derived facts strategies* in the test did not always result in accuracy. Further, Group 3 students did not always use *derived facts strategies* in interim calculations for the mental computation tasks. In fact, they often used *count* .

Efficient mental strategies (e.g., *wholistic* and *aggregation* - see Table 2) required good numeration understanding. Lower level alternative mental strategies (e.g., *separation left to right* ) also required numeration understanding (canonical and noncanonical). Some numeration understanding was also required for procedural understanding of *mental image of pen and paper algorithm* . This was the case with Group 2 students who were accurate, but not flexible. Group 4 students who were neither flexible nor accurate had very poor numeration understanding.

It has been posited (e.g., Reys, 1992; Sowder, 1988, 1994) that an understanding of the effects of operation on number would be important for efficient mental computation. In particular, understanding of the concepts of the effect of changing the addend and subtrahend would affect the ability to employ some *wholistic* strategies. Students (in the present study) who exhibited these number and operation understandings tended to employ high-level strategies (e.g., *wholistic compensation* ), whereas, students who did not exhibit these understandings, could not successfully complete examples using these strategies, although they attempted to access the strategies with scaffolding. It appeared that both numeration and number and operation understanding was required for successful employment of *wholistic* mental strategies.

In contrast to the findings of Reys, Bestgen, Rybolt, and Wyatt (1982), computational estimation did not support mental computation. Even proficient mental computers did not exhibit proficiency in computational estimation. One reason could be the students were too young to have developed estimation strategies. Heirdsfield (1996) found that some Year 4 students had developed some appropriate estimation strategies, and that these strategies were probably developed outside the classroom. It is possible that students in Year 3 are simply too young. Another reason could be the absence of estimation in the Year 3 syllabus (Department of Education, Queensland, 1991).

It has been posited that metacognition aids skilled mental computers (e.g., McIntosh et al., 1992; Sowder, 1994). However, in the present study metacognition did not feature strongly. The reason might lie in the young age of the students; they may be unable to distinguish their metacognitive knowledge in particular. On the other hand, they did seem to be able to verbalise their metacognitive beliefs (perceptions of their abilities). Metacognitive skills were important for flexibility in mental computation. Flexible mental computers, particularly Group 1 students, showed evidence of monitoring and checking.

Exceptional short-term recall and retention were not necessary for mental computation; however, threshold levels were necessary. These findings support those of Hunter (1978). Further, Hunter stated,

> *The expert (mental calculator) goes quite a way to meet these demands (of working memory), partly by the speed and quality of working, and partly by devising calculative methods which evade an excess of interrupted working.* (p. 343)

In the present study, flexible and accurate mental computers employed efficient mental strategies to alleviate demands on working memory. However, Group 2 students (accurate and not flexible) resorted to an automatic strategy (*mental image of pen and paper algorithm* ). Amy (Group 1) also reported using automatic strategies, but these strategies included a variety of efficient strategies. In contrast, students in Group 4 possessed poor recall, retention and executive functioning (i.e., poor working memory resources). They also had a poor knowledge base (in LTM). Further studies are needed to determine if poor working memory resources contribute to poor connections in LTM, resulting in diminished performance.

The results for Digit Span Test indicated that for most students (except Group 4) the phonological loop (PL) could support retrieval of number facts from LTM, and holding and rehearsal of interim calculations (of which there were many). However, Group 3 students did not have number facts in LTM, so the PL could not retrieve these. Further, there was evidence that the VSSP supported some strategies. The visual representation of the pen and paper algorithm, including interim calculations was stored and manipulated in the VSSP. However, there was little evidence of the use of the VSSP for Group 1 students. Although it was expected that numbers would be represented in some visual form, no Group 1 student reported this. The reason might lie in the young age of the students. They might have been unaware of their use of any mental imagery, or they may have been so preoccupied with their strategies, that they could not remember using any mental imagery.

Integration and compensation

Proficiency in mental computation (accurate and efficient mental strategies) required integrated understanding of number facts (speed, accuracy, and efficient number facts strategies when facts could not be automatically recalled), numeration, and number and operation. Proficient students also exhibited some metacognitive strategies and possessed reasonable short-term memory and executive functioning.

Where there was less knowledge and fewer connections between knowledge, students compensated in different ways, depending on their beliefs and what knowledge they possessed. One choice was to employ teacher taught strategies in which strong beliefs were held, as long as the procedures could be followed, and if they were supported by fast and accurate number facts and some numeration understanding (as in Group 2). Further, working memory (slave systems and central executive) had been sufficient. In particular, there was evidence of employment of the VSSP as a visual memory aid.

Another form of compensation was inventing strategies (as in Group 3) when the teacher taught strategies could not be followed. Although working memory was sufficient, the knowledge base was minimal and disconnected (in particular, number facts were not well known), thus compensation strategies were not efficient, and resulted in errors. Further, the knowledge base did not support high-level strategies. Some numeration understanding and sufficient memory (including executive functioning) supported the development of some alternative strategies, but no high level strategies. Access to *wholistic* strategies was only partially successful.

Finally, students who exhibited deficient and disconnected understanding (Group 4) tried to compensate by using teacher-taught procedures, but they were unsuccessful, as they also lacked procedural understanding and had poor memory (including diminished executive functioning).

Mental computation model

It appears there are four steps to mental computation when problems are presented:

1. recognise the numbers and operation involved,
2. select a strategy,
3. implement the strategy to arrive at a solution, and
4. check the solution.

All students in the study were capable of recognising the numbers and operation involved in the calculations. Factors involved in both selecting and implementing the strategy included access to and utilisation of facts, skills, strategies, and support from memory. Finally, it would be expected that students would check their solutions (although checking may occur during calculation as well). After analysing the results of the students it became evident that different students, in particular, different groups of students utilised different aspects of the framework. The "ideal" framework was formulated after identifying essential and threshold knowledge that supported accuracy in mental computation and flexibility in mental computation. Thus, the framework for Group 1 students who were accurate and flexible was the "ideal framework" (Figure 1).

## Limitations to the study

Because only two schools were approached to participate in this study, findings are not generalisable to all eight and nine year olds. Further, the two schools served similar socio-economic areas (although not geographical areas). Socio-economic factors could have had a great deal of influence on contextual factors, such as parental expectations and support. It was evident that most parents from the two schools expected their students to achieve.

Another limitation of this study was that only one obvious representative of Group 2 was interviewed in depth; therefore, this category might be artificial. There were a number of students in the pilot study, who were identified as Group 2 (4 out of 15 students); however, only one student participated in the in-depth interviews (as with other groups in the pilot study). Why there were more Group 2 students in the pilot study is not clear. It could be posited that to be accurate in mental computation, more efficient strategies than *mental image of pen and paper algorithm* need to be employed. It is possible that high accuracy when using pen and paper algorithms might require superior short-term recall and retention. This cannot be confirmed from the present study, as the only obvious Group 2 representative (Mandy) was not tested for memory, as memory was not a factor that was investigated at this stage.

Finally, there seemed to be difficulties eliciting some information from the students because of their age. Metacognitive knowledge did not appear to be present. Why strategies were chosen could not be elicited from the students. Further, students who employed high-level strategies did not report using any visual imagery, yet it would seem feasible that visual imagery would have supported these strategies.

## Implications for teaching

There was evidence of the importance of connected knowledge, including domain specific knowledge, and metacognitive strategies for proficient mental computation. This demonstrated the need for teaching practices to focus on the development of an extensive and integrated knowledge base to develop understanding; that is, concepts, facts, and strategies should not be learnt in isolation.

Students can and do formulate their own strategies (but not always accurately). However, invented procedures were more accurate and showed more number sense than teacher taught strategies. Therefore, students should be encouraged to formulate their own strategies. Because of memory load involved in mental computation, students should be permitted to use external memory aids (e.g. pen and paper). Further, mental strategies can be used to solve pen and paper exercises. It is posited that efficient mental strategies are also efficient written strategies. Pen and paper should be used as external memory aids for "jotting down" interim calculations and other scribbling, for instance, the empty number line. The empty number line has been successfully used in various Dutch studies (e.g., Selter, 1995). Selter had the students use the empty number line as a tool for thinking, and to reflect on and discuss their solution methods. Finally, by having students formulate their own computational algorithms, they have to call upon their own knowledge of numeration, number facts, etcetera; thus they develop connected knowledge while they develop their algorithms. This is in contrast to students using teacher-taught procedures, which require little connected knowledge.

Knowing number facts by immediate recall also decreases memory. Students in Year 3 were not expected to know number facts by recall. Heirdsfield (1996) found that even Year 4 students did not always know number facts by recall. Just as many Year 4 students calculated number facts by using efficient *DFS* , many Year 3 students in the present study also used *DFS* when they could not recall number facts. However, this caused extra load on working memory (at times). Therefore, permitting students to use pen and paper would alleviate working memory load due to lack of number fact knowledge. Further, students should be encouraged to build on their intuitive understandings, and should not be taught number facts strategies as drilled procedures (Van de Walle & Watkins, 1993). Moreover, students should be encouraged to choose their own appropriate number facts strategies, as not every student uses the same strategy (Gravemeijer, 1994). Finally, the development of number facts strategies (derived facts strategies) promotes the development of mental computation strategies.

Although some students were able to use the traditional pen and paper algorithms successfully, considering the amount of time that is spent in teaching these procedures, it appears that time could be better spent in having students develop their own strategies.

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